

Polarization-sensitive propagation in an anisotropic metamaterial with double-sheeted hyperboloid dispersion relation

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Abstract

The polarization-sensitive propagation in the anisotropic metamaterial (AMM) with double-sheeted hyperboloid dispersion relation is investigated from a purely wave propagation point of view. We show that TE and TM polarized waves present significantly different characteristics which depend on the polarization. The omnidirectional total reflection and oblique total transmission can occur in the interface associated with the AMM. If appropriate conditions are satisfied, one polarized wave exhibits the total refraction, while the other presents the total reflection. We find that the opposite amphoteric refractions can be realized by rotating the principle axis of AMM, such that one polarized wave performs the negative refraction, while the other undergoes positive refraction. The polarization-sensitive characteristics allow us to construct two types of efficient polarizing beam splitters under certain achievable conditions.

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I. INTRODUCTION

Media with negative permittivity and permeability have not been found in violation of any fundamental physical principles have been verified both experimentally and theoretically [1, 2, 3, 4, 5]. In such materials, the directions of energy transfer and wavefront propagation are opposite. This leads to remarkable electromagnetic properties such as refraction at surfaces that is described by a negative refraction index. It was also found negative refraction can occur at an interface associated with an anisotropic metamaterial (AMM), which does not necessarily require that all tensor elements of ϵ and μ have negative values [6, 7, 8, 9]. In general, TE and TM polarized waves propagate in different directions in an anisotropic medium. For a conventional anisotropic medium, all tensor elements of permittivity ϵ and permeability μ are positive. Wave propagation in conventional anisotropic crystal is an interesting topic with both a conceptual and a practical value. A large number of optical devices are based on the anisotropic effect, such as polarizers, compensators, switches etc., are currently employed in a large amount of experimental situations [10]. Recently, a broad range of applications have been suggested, such as partial focus lens [11, 12, 13, 14], spatial filters [15, 16], and polarizing beam splitters [17, 18] can be realized by AMMs.

In classic electrodynamics, it is well known that the three dimensional (3D) frequency contour of anisotropic materials is the combination of sphere and ellipsoid [19, 20]. Since the advent of negative media parameters in AMMs, the 3D frequency contour are significantly different from conventional anisotropic media. The corresponding wave-vector surfaces are a combination of ellipsoid or single-sheeted hyperboloid or double-sheeted hyperboloid. In the development of AMMs, there are several important questions easily be inquired: how the waves behave in AMMs, what characteristics may be useful for practical applications, and how to construct such AMMs. The importance of investigating AMMs with new wave-vector surfaces and its potential applications becomes evident when one considers that low-loss optical metamaterials are increasingly possible [21, 22, 23, 24]. Here we will focus our attention on the case that both TE and TM polarized waves exhibit double-sheeted hyperboloid wave-vector surfaces.

In this work, we want to present an investigation on the polarization-sensitive characteristics in the AMM with double-sheeted hyperboloid dispersion relation. First, we want to explore the wave propagation in the AMM with different combinations of tensor elements. We show that the omnidirectional total reflection and oblique total transmission can occur

in the interface associated with AMM. If certain conditions are satisfied, one polarized wave exhibits the total refraction, while the other presents the total reflection. Next, we concentrate our interest on the amphoteric refractions. We find that the opposite amphoteric refractions can be realized by rotating the principle axis of AMM, such that one polarized wave performs negative refraction, while the other undergoes positive refraction. Finally, we study the practical applications of the polarization-sensitive characteristics, and two types of polarizing beam splitters can be constructed.

II. WAVE PROPAGATION IN ANISOTROPIC METAMATERIALS

To reveal the phenomenon of polarization-sensitive propagation, we start with a purely 3D wave propagation point of view. It is currently well accepted that a better model is to consider anisotropic constitutive parameters, which can be diagonalized in the coordinate system collinear with the principal axis of the metamaterial [8, 9]. If we take the principal axis as z axis, the permittivity and permeability tensors have the following forms:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad \boldsymbol{\mu} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}, \quad (1)$$

where ε_i and μ_i are the permittivity and permeability constants in the principal coordinate system ($i = x, y, z$). In general, it is not enough for a complex metamaterial be characterized by six tensor elements, since the response of anisotropic metamaterial can be very complex. But for a special case, however, it is enough. Since we restrict the wave propagation at the $x - z$ plane. For a certain polarized wave, the propagation only decided by certain three parameters, other three parameters do not intervene.

Consider the propagation of a planar wave of frequency ω as $\mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$ and $\mathbf{H} = \mathbf{H}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$, through a regular isotropic medium toward an AMM. In isotropic media, the accompanying dispersion relation has the familiar form

$$k_x^2 + k_y^2 + k_z^2 = \varepsilon_I \mu_I \frac{\omega^2}{c^2}. \quad (2)$$

Here k_i is the i component of the incident wave vector, c is the speed of light in vacuum, ε_I and μ_I are the permittivity and permeability, respectively. Electromagnetic waves can be classified into two types: TE and TM waves. For a TE wave, its electric field is perpendicular

to the plane of incidence. For a TM wave, its magnetic field is normal to the plane of incidence. In isotropic media, both TE and TM waves exhibit the same dispersion relation. A careful calculation of Maxwell's equations in anisotropic media gives the dispersion relations:

$$\frac{q_x^2}{\varepsilon_y \mu_z} + \frac{q_y^2}{\varepsilon_x \mu_z} + \frac{q_z^2}{\varepsilon_y \mu_x} - \frac{\omega^2}{c^2} = 0, \quad (3)$$

$$\frac{q_x^2}{\varepsilon_z \mu_y} + \frac{q_y^2}{\varepsilon_z \mu_x} + \frac{q_z^2}{\varepsilon_x \mu_y} - \frac{\omega^2}{c^2} = 0, \quad (4)$$

for TE and TM polarized waves, respectively [25]. Here q_i represents the i component of transmitted wave-vector. The above equations can be represented by two three-dimensional surfaces in wave-vector space. It is well known that the wave propagation behaviors in anisotropic media are significantly different from those in isotropic materials [19, 20]. In general, anisotropic media exhibit a polarization-sensitive effect, namely TE and TM polarized waves present different characteristics which depend on the polarization. Here we are particularly interested in the case that one polarized wave exhibit total refraction, while the other present total reflection. Furthermore, we will discuss that the two polarized waves experience an opposite amphoteric refraction, such as polarized wave performs negative refraction, while the other undergoes positive refraction.

First we explore the two polarized waves show anomalous total refraction or total reflection. Without loss of generality, we assume the wave vector locate at the $x - z$ plane ($k_y = q_y = 0$). The incidence angle of light is given by $\theta_I = \tan^{-1}[k_x/k_z]$. Based on the boundary condition, the tangential components of the wave vectors must be continuous, i.e., $q_x = k_x$. Then the refraction angle of the transmitted wave vector or phase of TE and TM polarized waves can be written as $\beta_P^{TE} = \tan^{-1}[q_x/q_z^{TE}]$ and $\beta_P^{TM} = \tan^{-1}[q_x/q_z^{TM}]$, respectively. It should be noted that the actual direction of light is defined by the time-averaged Poynting vector $\mathbf{S} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$ [26]. The refraction angle of Poynting vector can be obtained as $\beta_S^{TE} = \tan^{-1}[S_{Tx}^{TE}/S_{Tz}^{TE}]$ and $\beta_S^{TM} = \tan^{-1}[S_{Tx}^{TM}/S_{Tz}^{TM}]$ for TE and TM waves, respectively.

In principle, the occurrence of transmission requires that the z component of the wave vector must be real. Setting $q_z = 0$ in Eq. (3) and Eq. (4), we can obtain the critical angles as

$$\theta_C^{TE} = \sin^{-1} \left[\sqrt{\frac{\varepsilon_y \mu_z}{\varepsilon_I \mu_I}} \right], \quad \theta_C^{TM} = \sin^{-1} \left[\sqrt{\frac{\varepsilon_z \mu_y}{\varepsilon_I \mu_I}} \right]. \quad (5)$$

If numerator larger than denominator inside the square root, no wave can transmitted for any incident angle, and the AMM will exhibit the interesting phenomenon of omnidirectional

total reflection. Here the omnidirectional total reflection means that the total reflection occurs for wave incidents at any angles. In addition, the oblique total reflection can not exist for a certain material parameters which make the corresponding expressions inside the square root negative.

The transmitted wave in AMMs can be determined by the two principles [27]: First, the boundary conditions require that the tangential component of the wave vector, is conserved across the interface, $q_x = k_x$. Second, energy conservation law requires that the energy current of the refracted waves should transmit away from the interface, i.e., the normal component of the Poynting vector, $S_{Tz} > 0$. For TE polarized wave, the transmitted Poynting vector is given by

$$\mathbf{S}_T^{TE} = Re \left[\frac{T_{TE}^2 E_0^2 q_x^{TE}}{2\omega\mu_z} \mathbf{e}_x + \frac{T_{TE}^2 E_0^2 q_z^{TE}}{2\omega\mu_x} \mathbf{e}_z \right]. \quad (6)$$

Analogously, for TM polarized wave, the transmitted Poynting vector is given by

$$\mathbf{S}_T^{TM} = Re \left[\frac{T_{TM}^2 H_0^2 q_x^{TM}}{2\omega\varepsilon_z} \mathbf{e}_x + \frac{T_{TM}^2 H_0^2 q_z^{TM}}{2\omega\varepsilon_x} \mathbf{e}_z \right]. \quad (7)$$

Here T_{TE} and T_{TM} are the transmission coefficient for TE and TM polarized waves, respectively. Based on the boundary condition, we can obtain the following expression for the transmission coefficients:

$$T_{TE} = \frac{2\mu_x k_z}{\mu_x k_z + \mu_I q_z^{TE}}, \quad T_{TM} = \frac{2\varepsilon_x k_z}{\varepsilon_x k_z + \varepsilon_I q_z^{TM}}. \quad (8)$$

Simultaneously, the corresponding reflection coefficients can be obtained as

$$R_{TE} = \frac{\mu_x k_z - \mu_I q_z^{TE}}{\mu_x k_z + \mu_I q_z^{TE}}, \quad R_{TM} = \frac{\varepsilon_x k_z - \varepsilon_I q_z^{TM}}{\varepsilon_x k_z + \varepsilon_I q_z^{TM}}. \quad (9)$$

It should be noted that the oblique total refraction emerges when a wave incident at Brewster angle. Mathematically the Brewster angles can be obtained from $R_{TE} = 0$ and $R_{TM} = 0$. In the next section, we will pay our attention to the case that TE polarized wave experiences oblique total refraction, while TM polarized wave undergoes total reflection, i.e. $R_{TE} = 0$. The anomalous wave propagation depends on the choice of the anisotropic parameters. In general, the critical angle is larger than the Brewster angle in regular anisotropic media. In present AMM, however, the situation is reversed. Here the anomalous wave propagation means that the Brewster angle is larger than the critical angle.

Next we want to study the opposite amphoteric refractions. Unlike in isotropic media, the Poynting vector in AMMs is neither parallel nor antiparallel to the wave vector, but

rather makes either an acute or an obtuse angle with respect to the wave vector [6]. In general, to distinguish the positive and negative refractions in AMMs, we must calculate the direction of the Poynting vector with respect to the wave vector. Positive refraction means $\mathbf{q}_x \cdot \mathbf{S}_T > 0$, and anomalous negative refraction suggests $\mathbf{q}_x \cdot \mathbf{S}_T < 0$. From Eqs. (6) and (7) we get

$$\mathbf{q}_x^{TE} \cdot \mathbf{S}_T^{TE} = \frac{T_{TE}^2 E_0^2 (q_x^{TE})^2}{2\omega\mu_z}, \quad \mathbf{q}_x^{TM} \cdot \mathbf{S}_T^{TM} = \frac{T_{TM}^2 H_0^2 (q_x^{TM})^2}{2\omega\varepsilon_z}. \quad (10)$$

We can see that the amphoteric refractions will be determined by μ_z for TE polarized wave and ε_z for TM polarized wave. The underlying secret of the opposite amphoteric refractions is that ε_z and μ_z always have the opposite signs. Evidently, we can choose an appropriate combinations of the tensor elements to realize the opposite amphoteric refractions. While we are particularly interested in the case that the amphoteric refractions present in the AMM with rotating principle axis. In the section IV, we will discuss such an interesting phenomenon in details.

III. DOUBLE-SHEETED HYPERBOLOID DISPERSION RELATION

In this section, we will explore the polarization-sensitive propagation in three types of AMMs. It should be noted that, for the same dispersion relation there exist two subtypes 3D wave-vector surface which can be formed from combinations of tensor elements. In fact, the two subtypes can be discussed in similar way. Hence we do not wish to get involved in the trouble to discuss every subtypes in detail.

Case I. The two double-sheeted hyperboloid have the same revolution axis. Here we choose the revolution axis coincide with z axis. The corresponding combination is chosen as $\boldsymbol{\varepsilon} = [+ , + , -]$ and $\boldsymbol{\mu} = [+ , + , -]$. The 3D frequency contour is plotted in Fig. 1(a). To investigated the propagating behaviors in this kind of AMM, we plotted the 2D refraction diagram in Fig. 1(b). The circle and the double-sheeted hyperbola represent the dispersion relations of isotropic regular media and AMMs, respectively. Both TE and TM polarized waves occur in the branch $-\pi/2 < \theta_I < \pi/2$. For the two polarized waves, $\mathbf{k}_z \cdot \mathbf{q}_z > 0$ and $\mathbf{q}_x \cdot \mathbf{S}_T < 0$, so the wave-vectors exhibit positive refractions, whereas Poynting vectors undergo negative refractions.

Case II. The two double-sheeted hyperboloid have the same revolution axis. Here we choose the revolution axis coincide with x axis. The corresponding combination is given by $\boldsymbol{\varepsilon} = [- , + , +]$ and $\boldsymbol{\mu} = [- , + , +]$. The 3D frequency contour is plotted in Fig. 1(a). To

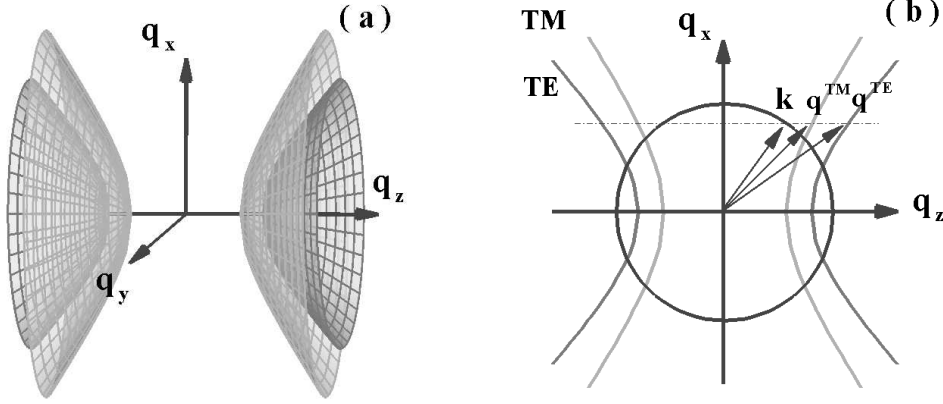


FIG. 1: The two double-sheeted hyperboloid have the same revolution axis. We assume the revolution axis coincide with z axis. (a) The 3D frequency contours present propagation characteristics of TE (black) and TM (gray) waves. (b) The circle and the double-sheeted hyperbola represent the dispersion relations of isotropic regular media and anisotropic metamaterial, respectively.

investigated the propagating behaviors, we depict the 2D refraction diagram in Fig. 1(b). For the two polarized waves, $\mathbf{k}_z \cdot \mathbf{q}_z < 0$ and $\mathbf{q}_x \cdot \mathbf{S}_T > 0$, so their refractions of wave vectors are negative, while the refractions of Poynting vectors are always positive. For TE polarized waves, if $\varepsilon_z \mu_y < \varepsilon_I \mu_I$ the propagation occur in the branch $-\pi/2 < \theta_I < -\theta_C^{TE}$ and $\theta_C^{TE} < \theta_I < \pi/2$. Note that the oblique total refraction can occur in the branch $-\theta_C^{TE} < \theta_I < \theta_C^{TE}$. If $\varepsilon_z \mu_y > \varepsilon_I \mu_I$, no wave can transmitted for any incident angle, and the AMM will exhibit the interesting phenomenon of omnidirectional total reflection. For TM polarized waves, if the inequality $\varepsilon_z \mu_y < \varepsilon_I \mu_I$, the propagation occurs in the branch $-\pi/2 < \theta_I < -\theta_C^{TM}$ and $\theta_C^{TM} < \theta_I < \pi/2$. If $\varepsilon_z \mu_y < \varepsilon_I \mu_I < \varepsilon_y \mu_z$, when $\theta_C^{TE} < \theta_I < \theta_C^{TM}$, only the TE polarized wave can propagate into the AMM for a certain incidence branch, while the TM polarized wave is totally reflected for any incidence angle.

Case III. The revolution axes of the two double-sheeted hyperboloid are perpendicular to each other. As an example, we want to explore the combination with $\boldsymbol{\varepsilon} = [+ , + , -]$ and $\boldsymbol{\mu} = [+ , - , -]$. The 3D frequency contour is plotted in Fig. 3(a). For TE polarized wave, $\varepsilon_y \mu_z$ is negative, thus the inequality $\varepsilon_y \mu_z < \varepsilon_I \mu_I$ satisfied for any incidence angle. In this case, the real wave vector exists for the branch $-\pi/2 < \theta_I < \pi/2$. Here $\mathbf{k}_z \cdot \mathbf{q}_z^{TE} > 0$ and $\mathbf{q}_x^{TE} \cdot \mathbf{S}_T^{TE} < 0$, the refraction of Poynting vector is always negative, even if the refraction of wave vector is always positive. For TM polarized waves, $\varepsilon_z \mu_y$ is positive, if $\varepsilon_z \mu_y < \varepsilon_I \mu_I$, the propagation occur in the branch $-\pi/2 < \theta_I < -\theta_C^{TM}$ and $\theta_C^{TM} < \theta_I < \pi/2$. Here

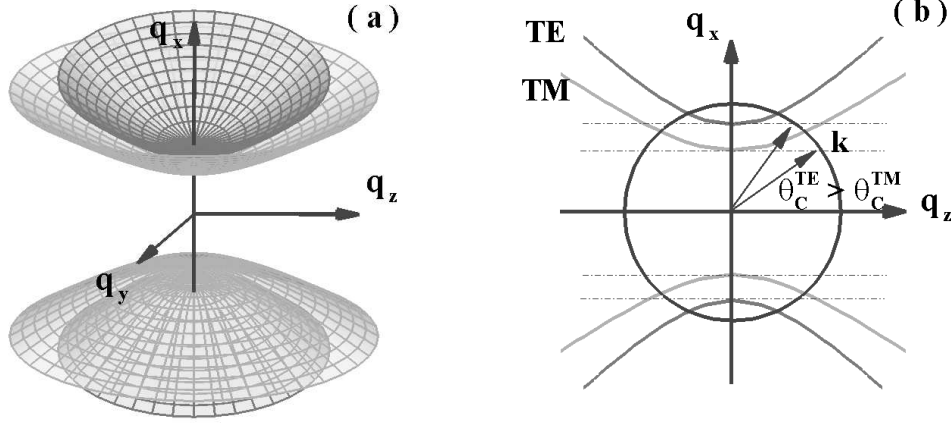


FIG. 2: The two double-sheeted hyperboloid have the same revolution axis. TE and TM waves have the same revolution axis which coincide with x axis. (a) The 3D frequency contours present propagation characteristics of TE (black) and TM (gray) waves. (b) The circle and the double-sheeted hyperbolas represent the dispersion relations of isotropic regular media and anisotropic metamaterial, respectively.

$\mathbf{k}_z \cdot \mathbf{q}_z^{TM} > 0$ and $\mathbf{q}_x^{TM} \cdot \mathbf{S}_T^{TM} < 0$, the Poynting vector exhibits negative refraction, while the wave-vector presents positive refraction. If $\varepsilon_z \mu_y > \varepsilon_I \mu_I$, the transmission phenomenon never occur in any incidence angles. From the above analyses, we can easily find that TE polarized wave exhibits the total transmission, while TM polarized wave presents total reflection.

To the best of our knowledge, the case III has not been discussed previously, since AMMs fall into only the following distinct groups: $\varepsilon_x = \varepsilon_y \neq \varepsilon_z$ and $\mu_x = \mu_y \neq \mu_z$ [7, 8]. In our case, however, there is no need for the elements satisfy such a relation. Thus we can reveal a new kind of wave-vector surface. In following analyses, we will pay more attention to the special case.

For the purpose of illustration, we summarize the amphoteric refractions and the corresponding incidence branch for TE and TM waves in Table I. Evidently, we can find that both TE and TM waves exhibit the same positive or negative refraction. In conventional anisotropic plasmas, only the tensor elements of permittivity could be permitted negative. Hence it is impossible for both TE and TM waves exhibit double-sheeted hyperboloid dispersion relation [19, 28]. In contrast to conventional anisotropic plasma, nonmagnetic AMMs have also been constructed recently [29, 30]. Since there is a distinct lack of free magnetic poles in the real world. The only way we can create a material with negative permeability is to fabricate [31]. In the present AMM, however, it is generally accepted tensor elements of

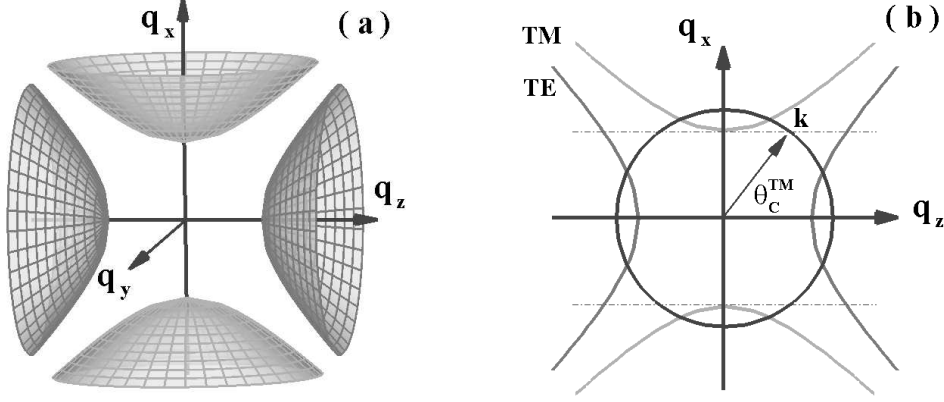


FIG. 3: The revolution axes of the two double-sheeted hyperboloid are perpendicular to each other. The revolution axis of TE and TM waves coincide with x and z axis, respectively. (a) The 3D frequency contours present propagation characteristics of TE (black) and TM (gray) waves. (b) The circle and the double-sheeted hyperbolas represent the dispersion relations of isotropic regular media and anisotropic metamaterial, respectively.

TABLE I: Amphoteric refractions and incidence branch for TE and TM waves. Note: X and Z indicate the revolution axes of wave-vector surface coincide with x and z axes. P and N denote positive and negative refraction, respectively.

| $\varepsilon_x \ \varepsilon_y \ \varepsilon_z$ | $\mu_x \ \mu_y \ \mu_z$ | TE Waves | TM Waves |
|---|-------------------------|--|--|
| $+$ $+$ $-$ | $+$ $+$ $-$ | Z N $[-\pi/2, \pi/2]$ | Z N $[-\pi/2, \pi/2]$ |
| $-$ $-$ $+$ | $-$ $-$ $+$ | Z P $[-\pi/2, \pi/2]$ | Z P $[-\pi/2, \pi/2]$ |
| $-$ $+$ $+$ | $-$ $+$ $+$ | X P $[-\pi/2, -\theta_C^{TE}] \cup [\theta_C^{TE}, \pi/2]$ | X P $[-\pi/2, -\theta_C^{TM}] \cup [\theta_C^{TM}, \pi/2]$ |
| $+$ $-$ $-$ | $+$ $-$ $-$ | X N $[-\pi/2, -\theta_C^{TE}] \cup [\theta_C^{TE}, \pi/2]$ | X N $[-\pi/2, -\theta_C^{TM}] \cup [\theta_C^{TM}, \pi/2]$ |
| $+$ $+$ $-$ | $+$ $-$ $-$ | Z N $[-\pi/2, \pi/2]$ | X N $[-\pi/2, -\theta_C^{TM}] \cup [\theta_C^{TM}, \pi/2]$ |
| $-$ $-$ $+$ | $-$ $+$ $+$ | Z P $[-\pi/2, \pi/2]$ | X P $[-\pi/2, -\theta_C^{TM}] \cup [\theta_C^{TM}, \pi/2]$ |
| $-$ $+$ $+$ | $-$ $-$ $+$ | X P $[-\pi/2, -\theta_C^{TE}] \cup [\theta_C^{TE}, \pi/2]$ | Z P $[-\pi/2, \pi/2]$ |
| $+$ $-$ $-$ | $+$ $+$ $-$ | X N $[-\pi/2, -\theta_C^{TE}] \cup [\theta_C^{TE}, \pi/2]$ | Z N $[-\pi/2, \pi/2]$ |

permittivity ε and permeability μ could be negative. Hence the present AMM will exhibit more interesting characteristics.

Now we want to enquire: what new application can be identified to utilize the polarization-sensitive wave characterizes. Here we introduce an idea to construct an effi-

cient polarizing beam splitter by using the AMM in the case III. To obtain a better picture of the effect of beam splitter, we consider a modulated beam of finite width. It should be pointed out that the modulated Gaussian beam has been extensively applied to investigated negative refraction [32, 33]. Following the method outlined by Lu *et al* [34], let us consider a modulate Gaussian beam with with squared magnitudes of TE and TM polarization incident from free space into the AMM slab. A general incident wave vector is written as $\mathbf{k} = \mathbf{k}_0 + \mathbf{k}_\perp$, where \mathbf{k}_\perp is perpendicular to \mathbf{k}_0 and $\omega_0 = ck_0$. We assume its Gaussian weight is

$$\tilde{E}(k_\perp) = \frac{w_0}{\sqrt{\pi}} \exp[-w_0^2 k_\perp^2], \quad (11)$$

where w_0 is the spatial extent of the incident beam. We want the Gaussian beam to be aligned with the incident direction defined by the vector $\mathbf{k}_0 = k_0 \cos \theta_I \mathbf{e}_x + k_0 \sin \theta_I \mathbf{e}_z$. For the purpose of illustration, the spatial maps of the electric fields are plotted in Fig. 4. We set the incidence angle equal to the Brewster angle $\theta_B^{TE} \simeq 35^\circ$, $w_0 = 2$, and $k_0 = 5$. We can easily find that TM polarized beam is totally transmitted, while TE polarized beam is totally reflected. The interesting characteristics allow us to construct a polarizing beam splitter.

Strictly speaking, the modulated Gaussian beam we used is not monochromatic. For a fundamental Gaussian beam, however, is monochromatic. From the point of Fourier optics, the monochromatic beam is considered to be composed of a series uniform plane waves travelling in sightly different directions. Therefore the beam splitting properties is still valid, since the monochromatic Gaussian only modulated in wave-vector space [35].

It should be mentioned that the amplification of evanescent waves could be achieved in the form of an AMM slab [7, 14]. Hence we can expect that both TE and TM evanescent waves can be amplified by the AMM slab. Here we want to concentrate our attention on the polarization-sensitive effect of propagating waves. Hence we do not involve in a detail discussion on evanescent waves. In addition, the backward wave propagation can occur in this type of AMM. To investigate the intriguing phenomena, we can calculate the direction of the Poynting vector with respect to the wave vector. The wave with $\mathbf{q} \cdot \mathbf{S}_T < 0$ has been called the backward wave or left-handed wave [6, 36, 37].

From the above analyses, we know that the AMM exhibit an intriguing polarization-sensitive propagation. Now we want to inquire: whether there is a kind of AMM in which TE and TM polarized waves propagate in the same direction. To investigate this question,

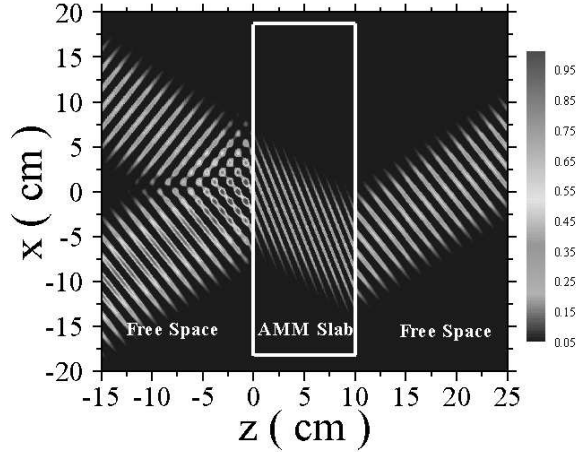


FIG. 4: The characteristics polarization-sensitive propagation in an AMM slab. We choose the AMM with combination of $\epsilon = [2, 1, -1]$ and $\mu = [2, -1, -1]$. The isotropic medium is assumed as vacuum with $\epsilon_I = 1$ and $\mu_I = 1$. TE polarized beam is totally transmitted, while TM polarized beam is totally reflected.

we can exam the transmission of wave-vector and Poynting vector. It is interested to note that if the tensor elements satisfied the following condition:

$$\frac{\epsilon_x}{\mu_x} = \frac{\epsilon_y}{\mu_y} = \frac{\epsilon_z}{\mu_z} = C \quad (12)$$

where C is a constant. If $C > 0$, the AMM will exhibit a polarization-insensitive propagation. In this case, both TE and TM polarized waves present the same wave-vector surface, such as a ellipsoid or a double-sheeted hyperboloid. The two polarized waves will exhibit the same propagating characteristic, and this AMM can be regard as quasi-isotropic [38]. If $C < 0$, TE and TM polarized waves also present the same single-sheeted hyperboloid wave-vector surface. However the two polarized waves will exhibit opposite amphoteric refractions [25].

From the table I, we have found that ϵ_z and μ_z always exhibit the same sign. Hence TE and TM waves will present the same positive or negative refraction. It is generally believed that the two polarized waves can not exhibit the opposite amphoteric refractions. Now a question naturally arise: how can the opposite amphoteric refractions can be realized in the AMM? In the following section, we want to explore this intriguing problem in detail.

IV. POLARIZATION-SENSITIVE PROPAGATION

The AMMs with rotating principle axis can exhibit some interesting physics phenomena, such as anomalous negative refraction [39, 40, 41], superluminal or subluminal group propagation [42], and large beam shift [43, 44]. Here we want to discuss the anomalous amphoteric refractions. We assume that there is an angle φ between the principle axis and the propagation axis. For TE polarized wave, the Maxwell's equations yield the dispersion relation in the AMM as

$$\alpha q_x^2 + \beta q_z^2 + \gamma q_x q_z = \frac{\omega^2}{c^2}. \quad (13)$$

Here q_x and q_z represent the x and z components of transmitted wave vector in the propagating coordinate system. The parameters α , β and γ are given by

$$\begin{aligned} \alpha &= \frac{1}{\varepsilon_y \mu_x \mu_z} (\mu_x \cos^2 \varphi + \mu_z \sin^2 \varphi), \\ \beta &= \frac{1}{\varepsilon_y \mu_x \mu_z} (\mu_x \sin^2 \varphi + \mu_z \cos^2 \varphi), \\ \gamma &= \frac{1}{\varepsilon_y \mu_x \mu_z} (\mu_z \sin 2\varphi - \mu_x \sin 2\varphi). \end{aligned} \quad (14)$$

The corresponding 3D dispersion geometry is shown in Fig. 5(a). We can find that it is a rotating manipulation of case III. The refraction diagram in $x - z$ plane is plotted in Fig. 5(b), where a plane electromagnetic wave is incident from free space into the AMM.

The z -component of the wave vector can be found by the solution of Eq. (13), which yields

$$q_z^{TE} = \frac{1}{2\beta} \left[\sigma \sqrt{4\beta \frac{\omega^2}{c^2} + (\gamma^2 - 4\alpha\beta)q_x^2} - \gamma q_x \right], \quad (15)$$

Here $\sigma = \pm 1$, the choice of sign ensures that light power propagates away from the surface to the $+z$ direction. The values of refraction wave-vector can be found by using the boundary condition and hyperbolic dispersion relation.

Now a question easily be asked: how to determine the positive or negative refraction in the special case? To distinguish the positive and negative refraction, we can calculate the direction of the Poynting vector with respect to the wave vector:

$$\mathbf{q}_x^{TE} \cdot \mathbf{S}_T^{TE} = \frac{4\varepsilon_y k_x k_z^2 (2\alpha k_x + \gamma q_z^{TE}) E_0^2}{(2k_z + 2\varepsilon_y \beta q_z^{TE} + \varepsilon_y \gamma k_x)^2}. \quad (16)$$

For TM polarized wave, S_T^{TM} can be obtained by exchanging ε_i and μ_i . Hence we can determine that TE and TM polarized waves exhibit the opposite amphoteric refractions in

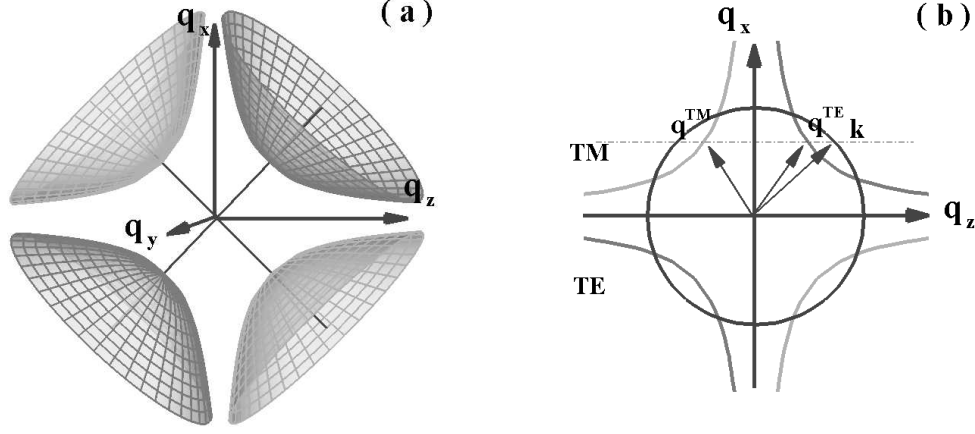


FIG. 5: We assume there is an angle φ between the principle axis and the propagation axis. (a) The 3D frequency contours present propagation characteristics of TE (black) and TM (gray) waves. (b) The circle and the double-sheeted hyperbolas represent the frequency contours of isotropic media and AMM, respectively.

such an AMM, such that TE polarized wave is positively refracted whereas TM polarized wave is negatively refracted. The opposite amphoteric refractions will result in a large birefringence.

Finally, we want to introduce another type of polarizing beam splitter, which is based on the opposite amphoteric refractions. For the purpose of illustration, the spatial maps of the electric fields are plotted in Fig. 6. We can choose the appropriate medium parameters, then the reflections of TE and TM polarized waves are completely absent. A large beam splitting angle between the two polarized waves can be obtained as 90° . Compared with the polarizing beam splitters made from the conventional anisotropic crystal, the present counterpart is more simple and more efficient. However the important limitation in practical realization of the splitter is loss in the AMM slab. We trust that it is advantageous to employ ultralow-loss AMMs to construct a very simple and very efficient splitter [18].

In the above analyses, the interesting effects of polarization-sensitive propagation in AMM are discussed using the 3D frequency contours. From a purely wave propagation point of view, the values of permittivity and permeability tensor elements were taken as constants at a fixed frequency. A question can then be asked in regard to the influence of frequency dispersion on the above analysis. In principle, the geometry of the wave-vector surface is determined by the signs of the medium parameters. In a certain frequency region where the permittivity and permeability tensor elements change signs, the corresponding wave-

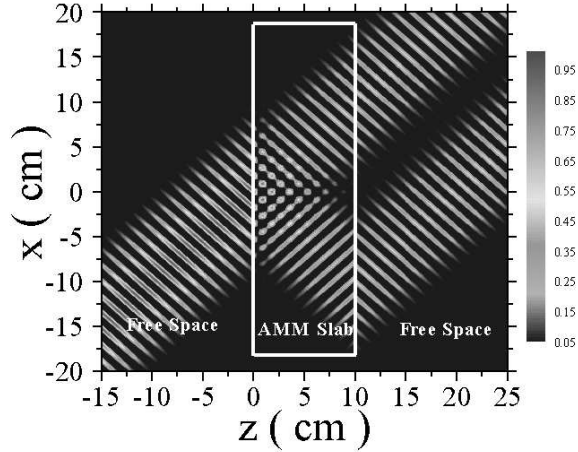


FIG. 6: The characteristics polarization-sensitive propagation in an AMM slab. We choose the AMM with combination of $\epsilon = [1, 1, -1]$ and $\mu = [1, -1, -1]$. The isotropic medium is assumed as vacuum with $\epsilon_I = 1$ and $\mu_I = 1$. TM polarized beam is negatively refracted, while TE polarized beam is positively refracted.

vector surfaces will present and a new characteristics of wave propagation emerges. Hence we expect our analyses can be extended to study the general behavior of other possible combinations.

V. CONCLUSION

In conclusion, we have investigated polarization-sensitive propagation in the AMM with double-sheeted hyperboloid dispersion relation. Under appropriate conditions, the anomalous omnidirectional total reflection and oblique total refraction can occur in the interface associated with the AMM. We are especially interested in the case that one polarized wave is totally refracted, the other is totally reflected. We have studied the opposite amphoteric refractions, such that one polarized wave exhibits positive refraction, while the other presents negative refraction. Based on the polarizing-sensitive propagation, we have introduced two types of polarizing beam splitters. We are sure that our scheme has not exhausted the interesting properties. The wave characteristics of polarization-sensitive propagation could be taken advantage of to other practical applications, such as polarizers, beam filters, and polarization dependent lenses.

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